

$$f(x) = x \ln x \quad g(x) = ae^x \quad (a \in \mathbb{R})$$

$$y = f(x) \quad x = 1$$

$$y = g(x) \quad a$$

$$G(x) = f(x) - g(x)$$

$$a \quad ae^2 - 2 \quad G(x) = 0$$

$$a = e^{-2} \quad a \quad \left(0, \frac{1}{e}\right)$$

$$F(x) = \frac{G(x)}{x} = \ln x - \frac{ae^x}{x} \quad F'(x) = \frac{x - a(x-1)e^x}{x^2}$$

$$0 \quad x \quad a \quad \frac{2}{e^2} \quad F'(x) = 0 \quad F(x) \quad (0,1)$$

$$F(x) \quad F(1) = -ae = 0$$

$$x \quad F'(x) = -\frac{a(x-1)}{x^2} \left[e^x - \frac{x}{a(x-1)} \right]$$

$$H(x) = e^x - \frac{x}{a(x-1)} \quad H'(x) = e^x + \frac{1}{a(x-1)^2} = 0$$

$$a \quad \frac{2}{e^2} \quad H(2) = \frac{ae^2 - 2}{a} = 0 \quad m \in (1,2) \quad \frac{m}{a(m-1)} = e^2$$

$$1 \quad m \quad \frac{ae^2}{ae^2 - 1} \quad H(m) = e^m - \frac{m}{a(m-1)} \quad e^2 - e^2 = 0 \quad H(m) \cdot H(2) = 0$$

$$H(x) \quad x_0 \in (1,2) \quad F(x) \quad x_0 \in (1,2)$$

$$H(x_0) = 0 \quad e^{x_0} = \frac{x_0}{a(x_0-1)} \quad F(x_0) = \ln x_0 - \frac{1}{x_0-1} \quad x_0 \in (1,2)$$

$$F'(x_0) = \frac{1}{x_0} + \frac{1}{(x_0-1)^2} = 0 \quad F(x_0) \quad (1,2)$$

$$F(x_0) \quad F(2) = \ln 2 - \frac{ae^2}{2} \quad \ln 2 - 1 = 0$$

$$ae^2 - 2 \quad \frac{G(x)}{x} = 0 \quad G(x) = 0$$

$$F(x) = \frac{G(x)}{x} = \ln x - \frac{ae^x}{x}$$

$$F'(x)$$

$$0 < x < 1 \quad F'(x) = 0 \quad F(x) = 0 \quad x > 1 \quad F(x)$$

$$F'(x) = -\frac{a(x-1)}{x^2} \left[e^x - \frac{x}{a(x-1)} \right] \quad H(x) = e^x - \frac{x}{a(x-1)} \quad H(x)$$

$$(1, 2) \quad x_0 \quad F(x_0) \quad F(x_0) = 0$$

$$G(x) = 0 \Leftrightarrow G(x)_{\max} = 0 \Leftrightarrow F(x)_{\max} = 0$$

x

$$ae^2 - 2 \quad a > \frac{2}{e^2} \quad G(x) = x \ln x - ae^x < 0 \quad a > \frac{x \ln x}{e^x}$$

$$(i) \quad 0 < x < 1 \quad \frac{x \ln x}{e^x} < 0$$

$$(ii) \quad x > 1 \quad h(x) = \frac{x \ln x}{e^x} \quad h'(x) = \frac{\ln x + 1 - x \ln x}{e^x}$$

$$u(x) = \ln x + 1 - x \ln x, (x > 1) \quad u'(x) = \frac{1}{x} - \ln x - 1$$

$$u'(x) < 0 \quad [1, +\infty) \quad u'(1) = 0 \quad u(x) < 0 \quad [1, +\infty)$$

$$u(2) = 1 - \ln 2 > 0 \quad u(3) = 1 - 2 \ln 3 < 0$$

$$\exists x_0 \in (2, 3) \quad h'(x_0) = 0 \quad \ln x_0 + 1 - x_0 \ln x_0 = 0$$

$$1 < x < x_0 \quad h'(x) > 0 \quad h(x) < h(x_0) \quad x > x_0 \quad h'(x) < 0 \quad h(x) < h(x_0)$$

$$h(x) < h(x_0) = \frac{x_0 \ln x_0}{e^{x_0}} = \frac{\ln x_0 + 1}{e^{x_0}}$$

$$\frac{\ln x_0 + 1}{e^{x_0}} < \frac{2}{e^2} \quad \ln x_0 < x_0 - 1, \quad y = \frac{x}{e^x} \quad 2, 3$$

$$(i) (ii) \quad a > \frac{x \ln x}{e^x}$$

$$a > \frac{x \ln x}{e^x} \quad \frac{2}{e^2} = \left(\frac{x \ln x}{e^x} \right)_{\min}$$

$$h(x) = \frac{x \ln x}{e^x}$$

$$h'(x) = 0 \quad x_0 \quad \text{max}$$

$$\frac{\ln x}{x} - \frac{2e^{x-2}}{x^2} \quad \varphi(x) = \frac{\ln x}{x} \quad \varphi(x) \quad (0, e) \quad x \ln x - 2e^{x-2}$$

$$(e, +\infty) \quad \varphi(x) \quad \varphi(e) = \frac{1}{e}$$

$$m(x) = \frac{2e^{x-2}}{x^2} \quad m'(x) = \frac{2e^{x-2}(x-2)}{x^3}$$

$$m(x) \quad (0, 2) \quad (2, +\infty)$$

$$m(x) \quad m(2) = \frac{1}{2} - \frac{1}{e} \quad \varphi(x) \quad \frac{\ln x}{x} - \frac{2e^{x-2}}{x^2}$$

$$ae^2 - 2 \quad x \ln x - 2e^{x-2}$$

$$\frac{\ln x}{x} - \frac{2e^{x-2}}{x^2} \quad \left(\frac{\ln x}{x}\right)_{\max} \quad \left(\frac{2e^{x-2}}{x^2}\right)_{\min}$$

$$\ln x - \frac{2e^{x-2}}{x} = 0 \quad \ln x = e^x$$

$$\varphi(x) = \frac{2e^{x-2}}{x} \quad x = 2 \quad x \ln x - 2e^{x-2} \quad \frac{2e^{x-2}}{x} \quad \ln x \quad y = \frac{1}{2}x \quad \frac{2e^{x-2}}{x} - \frac{1}{2}x$$

$$x = 2 \quad \frac{1}{2}x - \ln x \quad \frac{2e^{x-2}}{x} - \ln x$$

$$\varphi(x) = \frac{2e^{x-2}}{x} \quad g(x) = a(2 \ln x + 1)$$

$$f(x) = ae^x - \ln x - 1$$

$$x = 2 \quad f(x) \quad a \quad f(x)$$

$$a - \frac{1}{e} \quad f(x) = 0$$

$$f(x) = e^{2x} - a \ln x$$

$$f(x) \quad f'(x)$$

$$a > 0 \quad f(x) \quad 2a + a \ln \frac{2}{a}$$

$$f(x) = ae^x \ln x + \frac{be^{x-1}}{x} \quad y = f(x)$$

$$(1, f(1)) \quad y = e(x-1) + 2$$

$$a, b \quad f(x) > 1$$

$$f(x) = \frac{2 \ln x + 2}{e^x}$$

$$f(x)$$

$$x > 0 \quad f'(x) \ln(x+1) < \frac{2}{e^x} + \frac{2}{e^{x+2}}$$

$$f(x) \quad (0, 1) \quad (1, +\infty)$$

$$f'(x) \ln(x+1) < \frac{2}{e^x} + \frac{2}{e^{x+2}} \quad (1-x-x \ln x) \ln(x+1) < \left(1 + \frac{1}{e^2}\right)x$$

$$g(x) = 1 - x - x \ln x \quad g'(x) = -1 - (\ln x + 1) = -2 - \ln x$$

$$0 < x < \frac{1}{e^2} \quad g'(x) > 0 \quad x > \frac{1}{e^2} \quad g'(x) < 0$$

$$g(x) \quad \left(0, \frac{1}{e^2}\right) \quad \left(\frac{1}{e^2}, +\infty\right) \quad g(x) \quad 1 - \frac{1}{e^2} + \frac{2}{e^2} = 1 + \frac{1}{e^2}$$

$$1 - x - x \ln x \quad 1 + \frac{1}{e^2} \quad \ln(x+1) < x \quad x = 1$$

$$0 < \ln(x+1) < x \quad (1-x-x \ln x) \ln(x+1) < \left(1 + \frac{1}{e^2}\right)x$$

$$x > 0 \quad f'(x) \ln(x+1) < \frac{2}{e^x} + \frac{2}{e^{x+2}}$$

$$\ln x \quad e^x$$

$$\ln x \quad x-1 \Leftrightarrow e^x \quad x+1$$

$$x \quad e^{2x} - a \ln x \quad \frac{1}{2}a \quad a$$

$$2e^{2x} \quad a(2 \ln x + 1) \quad f(x) = 2e^{2x} \quad g(x) = a(2 \ln x + 1)$$

$$a > 0 \quad a > 0$$

$$a > 0$$

$$t$$

$$\begin{cases} f(t) = g(t) \\ f'(t) = g'(t) \end{cases} \begin{cases} 2e^{2t} = a(2\ln t + 1) \\ 4e^{2t} = \frac{2a}{t} \end{cases} \begin{cases} t = 1 \\ a = 2e^2 \end{cases}$$

$$a \quad [0, 2e^2]$$

$$e^{2x} a \left(\ln x + \frac{1}{2} \right) \quad e^{2x} a \ln(\sqrt{ex}) \quad f(x) = e^{2x} a \ln(\sqrt{ex}) - 0$$

$$a - 0 \quad a - 0$$

$$a - 0 \quad e^{2x} \frac{a}{2} \ln(\sqrt{ex})^2 \quad \frac{a}{2} \frac{e^{2x}}{\ln(ex^2)} \quad e^{2x} (e^x)^2 (ex)^2$$

$$\ln(ex^2) \cdot \frac{1}{e} \cdot ex^2 = x^2 \quad x = 1 \quad \left(\frac{e^{2x}}{\ln(ex^2)} \right)_{\min} = \frac{(ex)^2}{x^2} = e^2$$

$$\frac{a}{2} e^2 - 0 = a - 2e^2 \quad a \quad [0, 2e^2]$$

$$f(x) = 2e^{2x}$$

$$g(x) = a(2\ln x + 1)$$

$$\left(\frac{e^{2x}}{\ln(ex^2)} \right)_{\min}$$

$$e^x - ex \quad \ln x - \frac{x}{e}$$



